Non-Preemptive Fixed-Priority Uniprocessor Scheduling where the Execution Time of a Job Depends on the Scheduling of Jobs that Executed Before it

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Abstract— **We consider non-preemptive fixed-priority scheduling of a set of constrained-deadline sporadic tasks on a single processor. We assume that the execution time of a job J depends on the sequence of jobs executed before J, that is, the execution time of a job of a task is not a constant. We raise the following two open problems (i) given a priority assignment, can the response time of a task be computed in pseudo-polynomial time? and (ii) how to create an optimal priority-assignment scheme?**

I. Motivation

With today's processors, the execution time of a task is heavily dependent on whether a memory operation (load/store) results in a cache hit or a cache miss. Even with non-preemptive scheduling and even with a single processor, the execution time of a job of a task depends on the scheduling of other tasks. As an illustration, consider tasks τ_1, τ_2 and τ_3 which are scheduled non-preemptively on a single processor. Jobs of task τ_1 start their execution by reading a variable x, where x is an array. Ditto for the jobs of task τ_2: jobs of task τ_2 start their execution by reading a variable x. However, jobs of task τ_3 never access variable x. For each of the tasks τ_1 and τ_2, it holds that its jobs have the execution time 5 milliseconds if variable x was not in the cache. If a job of task τ_2 executed immediately after a job of task τ_1, then τ_1's job will experience a cache miss when referencing the variable x but we may be able to prove than when the job of task τ_2 references variable x, it results in cache hits when τ_2 references x and hence the execution time of τ_2's job becomes 4 milliseconds.

Therefore, we need a scheduling theory which takes into account the fact that the execution time of a job may depend on which jobs executed just before it. Unfortunately, the current research literature offers no such scheduling theory.

II. Model

Task and platform characterization. We consider a system comprising a single processor and a software system comprising a task set τ composed of n constrained-deadline sporadic tasks. A task $\tau_i \in \tau$ is characterized by integers D_i and T_i with the interpretation that the task generates a (potentially infinite) sequence of jobs where the arrival times of jobs by τ_i are separated by at least T_i time units and a job of task τ_i must finish its execution within D_i time units after its arrival.

The execution time of a job depends on the job executing before it, and we therefore define the following concepts for task τ_i. The symbol $nhistories_i$ is an integer greater than or equal to one. $historylength_i^h$ is an integer greater than or equal to one and it is defined for $1 \le h \le nhistories_i$. The symbol $historyitem_i^{h,k}$ is an integer in $\{1,2,3,\ldots,n\}$ and it is defined for $1 \le h \le nhistories_i$ and $1 \le k \le historylength_i^h$.

We say that the for job J generated by task τ_i, the execution time C_i^h is *historyallowed* if it holds for the $historylength_i^h$ jobs that executed before J that for each $j \in \{1,2,\ldots,historylength_i^h\}$, the jth job before J is the generated by the task with index $historyitem_i^{h,j}$. The execution time of a job is the minimum among all its *historyallowed* execution times. Note that C_i^h is *historyallowed* if $historylength_i^h=0$. We assume that for each task τ_i there is one h such that $historylength_i^h=0$.

Figure 1a shows an example task set in this model.

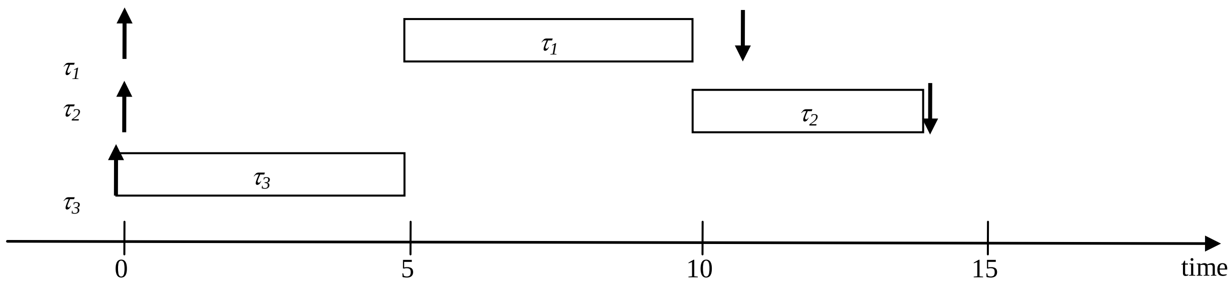
Scheduling. We assume that each task τ_i is assigned a priority $prio_i$ and each job generated by task τ_i is given the priority of the task that generated the job. We say that a job J is *eligible for execution* at time t, if (i) job J arrives at t or earlier and (ii) job J finishes execution later than t.

We assume non-preemptive scheduling, that is, if a job has started to execute then it will continue to execute until it finishes. When a job finishes, the job selected for execution is the one with the highest priority among the jobs that are eligible for execution at that time. Figure 1(b) and Figure 1(c) show examples of schedules.

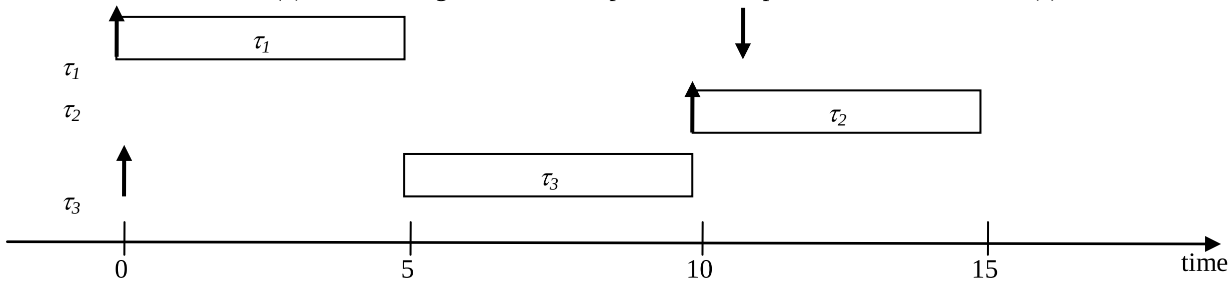
n=3,

T_1=50, D_1=11,	T_2=150, D_2=14,	T_3=500, D_3=500,
$nhistories_1$=2	$nhistories_2$=2	$nhistories_3$=1
C_1^1=5, $historylength_1^1$=0	C_2^1=5, $historylength_2^1$=0	C_3^1=5, $historylength_3^1$=0
C_1^2=4, $historylength_1^2$=1, $historyitem_1^{2,1}$=2	C_2^2=4, $historylength_2^2$=1, $historyitem_2^{2,1}$=1	

(a) An example of a task set. This task set models that task τ_1 and τ_2 share some variables and therefore executing one of them just before the other reduces the execution time of the other.



(b) A schedule generated for a specific arrival pattern for the task set in (a).



(c) A schedule generated for another specific arrival pattern for the task set in (a).

Figure 1. An example of a task set and two examples of schedules that can be generated for different arrival patterns.

Response time and schedulability. The response time of a job is the time that the job finishes execution minus the arrival time of the job. The response time of a task τ_i (denoted R_i) is the maximum response time that a job of τ_i can experience. We say that a task set is *schedulable* with respect to priority assignment *P* if $\forall i$: $R_i \leq D_i$. We say that a task set is *non-preemptive fixed-priority feasible* if there exists a priority assignment such that the task set is schedulable with respect to this priority assignment. We say that a priority-assignment scheme A is *optimal* if for each task set that is non-preemptive fixed-priority feasible, the task set is schedulable with respect to the priority given by the priority-assignment scheme A.

Note that in Figure 1(b), the job of task τ_2 has execution time of four time units because it executes after a job of task τ_1. Hence the job of task τ_2 meets its deadline. Classical non-preemptive analysis however, which does not consider that the execution time of a job depends on the job that executes before it, would calculate an upper bound on the response time being one time unit longer than the one in the example in Figure 1(b) and hence classical non-preemptive analysis would deem the task set in Figure 1(a) unschedulable.

Figure 1(c) shows an example of a schedule for another arrival pattern. Note here that the job by τ_2 does not execute directly after a job of task τ_1 and hence the execution time of the job of task τ_2 is five, that is, one time unit more than it was in Figure 1(b).

III. Open Problem Formulation

We propose the following two open problems:

> **OP1.** Is it possible to create an algorithm, with pseudo-polynomial time-complexity, which computes R_i?
>
> **OP2.** How to create an optimal priority assignment scheme.

We believe these two problems are interesting because both of them have (affirmative/positive) answers/solutions for the case that the execution time of a job does not depend on its history. But for our model they are unresolved.

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